Snell's law from Heisenberg's Uncertainty Principle

M.I.Mohamed
Department of Physics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

November 9, 2020

Abstract

In 1621, Willebrord Snell experimented with light passing through different media. He developed a relationship called Snell's law, which can be used to find the angle of refraction for light traveling between any two media.to derive the snell law we can use many ways like Fermat's principle , Maxwell's Equations ,Huygens's principle or conservation of energy and momentum. but in this paper we will use the Heisenberg's Uncertainty Principle to derive Snell's law .

1 Introduction

If light travels from one transparent medium to another at any angle other than straight on (normal to the surface), the light ray changes direction when it meets the boundary. As in the case of reflection, the angles of the incoming and refracted rays are measured with respect to the normal. For studying refraction, the normal line is extended into the refracting medium, this phenomenon was explained by Snell's law states that the ratio of the sines of the angles of incidence and refraction is equivalent to the ratio of phase velocities in the two media, or equivalent to the reciprocal of the ratio of the refraction indices

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{\nu_1}{\nu_2} \tag{1}$$

which θ as the angle measured from the normal of the boundary, υ as the velocity of light in the respective medium , and n as the refractive index of the respective medium.we can study this phenomenon by Uncertainty Principle to derive the snell's law becouse Heisenberg's uncertainty principle can be generalized to any pair of complementary, or canonically conjugate, dynamical variables: it is impossible to devise an experiment that can measure simultaneously two complementary variables to arbitrary accuracy . momentum and position, for instance, form a pair of complementary variables. and snell's law

include the velocity or refraction indices which any one of them related to momentum .

2 Method

Let refraction of light at the interface between two media of different refractive indices, with $n_2 \rangle n_1$ and the phenomenon will occure as the **figure(Nada) .**

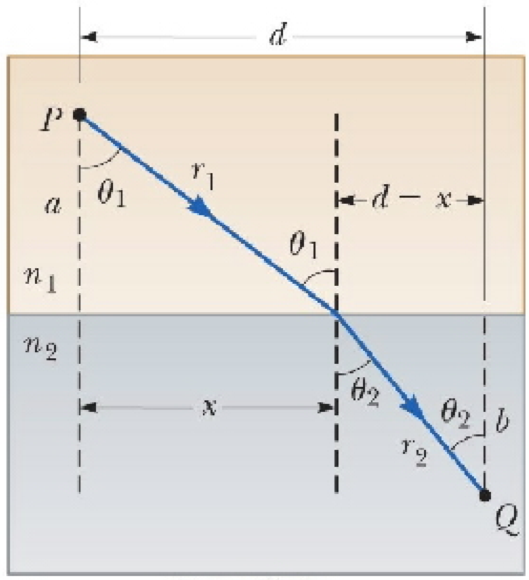

figure Nada

In the system 1, if we applied the uncertainty principle about the the system of photon (light) which we can write the total average uncertaninties of all the matter by summing up all the possible interaction between the planck units (n) in the bodies. then the total uncertainty

$$\Delta P_{r_1}\ \Delta r_1 \geq \sum_{i=1}^{n}\ \hbar \tag{2}$$

We will partly repeat that derivation here, but we also develop some important new insights. Heisenberg's uncertainty principle is given by

$$\Delta P_{r_1}\ \Delta r_1 \sim \sum_{i=1}^{n}\ \hbar \tag{3}$$

let N=$\sum_{i=1}^{n}$, since the non relativistic momentum $\Delta P_{r_1} = m\Delta\nu_{r_1}$

$$m\Delta\nu_{r_1}\ \Delta r_1 \sim N\ \ \hbar \tag{4}$$

From **figure(Nada)** $\Delta r = \frac{\Delta x}{\sin\theta_1}$

$$\frac{\Delta \nu_{r_1}}{\sin\theta_1} \sim \frac{N\ \hbar}{m\ \Delta x} \tag{5}$$

In system 2 , if we applied the uncertainty principle about the the system of photon which we can write the total average uncertanintes of all the matter by summing up all the possible interaction between the planck units (z) in the bodies. then the total uncertanity

$$\Delta P_{r_2}\ \Delta r_2 \geq \sum_{i=1}^{z}\ \hbar \tag{6}$$

$$\Delta P_{r_2}\ \Delta r_2 \sim \sum_{i=1}^{z}\ \hbar \tag{7}$$

let D=$\sum_{i=1}^{z}$, since the non relativistic momentum $\Delta P_{r_2} = m\Delta\nu_{r_2}$

$$m\Delta\nu_{r_2}\ \Delta r_2 \sim D\ \hbar \tag{8}$$

From **figure(Nada)** $\Delta r = \frac{\Delta(d-x)}{\sin\theta_2}$,but if we detect the photon the value of d not act to the value was measured , by other words the uncertanity of d (Δd) is equal to zero . so we get

$$\Delta r = \frac{\Delta x}{\sin\theta_2} \tag{9}$$

using Eq(8)

$$\frac{\Delta \nu_{r_2}}{\sin\theta_2} \sim \frac{D\ \hbar}{m\ \Delta x} \tag{10}$$

Now we assume the number of photons are constant which no absorbed of poton , this meaning D=N , if we use this in $Eq\,(5)$ and $Eq\,(10)$. so we get

$$\frac{\Delta \nu_{r_1}}{\sin\theta_1} \sim \frac{\Delta \nu_{r_2}}{\sin\theta_2} \tag{11}$$

which if $\Delta\nu_{r_1} \rightarrow \nu_{r_1} then\ \ \Delta\nu_{r_2} \rightarrow \nu_{r_2}$

$$\frac{\nu_{r_1}}{\sin\theta_1} \sim \frac{\nu_{r_2}}{\sin\theta_2} \tag{12}$$

to show the refraction indices should multiplication the two sides by $\frac{1}{c}$ becouse the defination of refraction indices is n=$\frac{c}{v}$, so we get

$$\frac{\sin\theta_1}{\sin\theta_2} \sim \frac{n_2}{n_1} \tag{13}$$

this is the snell's law .

3 conclusion

In this derivation we explained we can derive the snell's law from Heisenberg's uncertainty principle if and only if we assume the number of photos are constant , but if we use for example Beer–Lambert–Bouguer law or the correction from Haim Abitan, Henrik Bohr, and Preben Buchhave we can calculate the correction term in this form by the ratio between N and D , probability it wil be propertional with $e^{-\mu t}$,which μ is absorption coefficient , t is thickness of medium .

4 Acknowledgments

Thanks to Nabil Ibrahim Zaki El Qenawy for encouragement

5 Reference

[1] J. D. Jackson Classical electrodynamics, (Wiley, N.Y., 1999).

[2] G.L.J.A. Rikken and B.A. van Tiggelen, Phys. Rev. Lett. 108 (2012) 230402.

[3] Haim Abitan, Henrik Bohr, and Preben Buchhave ,Correction to the Beer-Lambert-Bouguer law for optical absorption

[4] W. Heisenberg.¨ Uber den anschaulichen inhalt der quantentheoretischen kinematik und mechanik.Zeitschrift f¨ur Physik,

(43):172–198, 1927

[5] Heisenberg W Zeits. f. Phys. 43 172 1927

[6] Hoffmann, D., 2001, On the Experimental Context of Planck's Foundation of Quantum Theory1: Centaurus, v. 43, no. 3-4, p. 240-259, 10.1111/j.1600-0498.2000.cnt430305.x.